Does Age Matter for the Labor Market Effects of Technological Progress?

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Outline of talk

1. Introduction
   - Our Questions, Motivation and Approach
   - Summary of Findings
2. Methodology
   - Conventional SVAR
   - Basics of Functional Analysis
   - Mixed Autoregression (MAR)
3. Results
   - SVAR with Aggregate Employment
   - MAR with Employment Curve
4. Conclusions
5. Extended MAR
   - Variable Selection by Adaptive LASSO
   - Results
6. Conclusions

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Our Questions

A central question in macroeconomics: How do labor market outcomes change when there is technological progress?

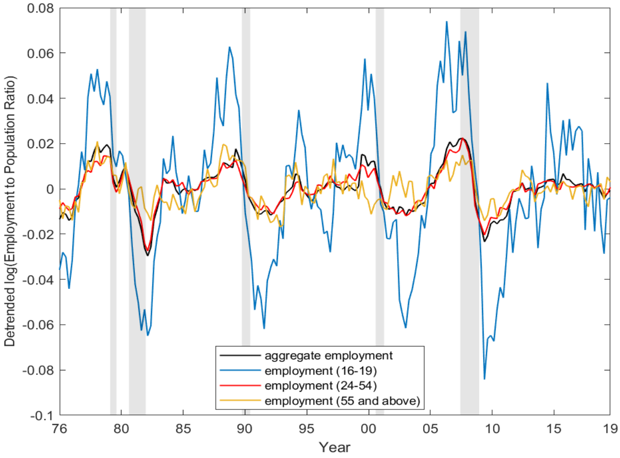
We broaden this question by asking if there exists age-specific effects.

Specifically, we ask the following questions:

- Does the employment across different ages respond to technology shocks heterogeneously in business cycles?
- Are technology shocks, or non-technology shocks, the main driving forces of the cyclical fluctuations of employment across different ages?

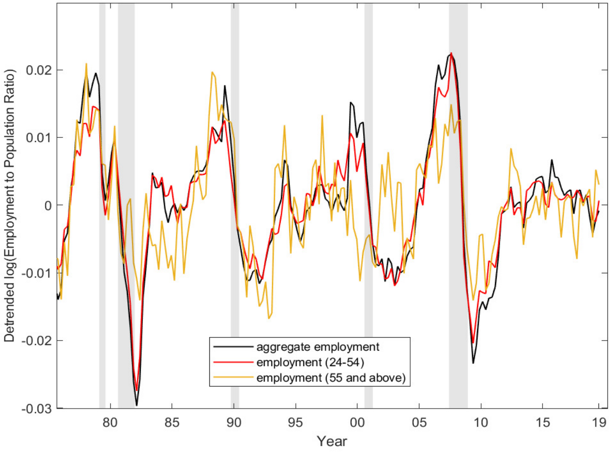
Motivating stylized facts are provided next.

Young, Prime-Age, Old vs. Aggregate Employment



Cyclical Movement of the Employment of Young, Prime-Age, Old: Detrended by an HP-filter with a smoothing parameter of 1600. In 2019, 13% are of ages 16-24, 69% of ages 25-54, and 18% of ages 55-64.

Prime-Age, Old vs. Aggregate Employment



Cyclical Movement of Employment of Prime-Age and and Old.

Literature on Aggregate Employment

Galí (1999) finds

- Aggregate employment (detrended employment level) responds to technology shocks negatively. ▸ data
- Employment is countercyclical, conditional on technology shocks

However, aggregate level analysis may smooth out important underlying individual level heterogeneities

Hence, although average employment responds negatively to a positive technology shock, this may not be true for all ages. Employment is quite heterogeneous across different ages in volatility and cyclicality, for example.

▸ More on Aggregate Literature

Literature on Disaggregate Employment

Clark and Summers (1978)

- Studies the responses of log employment to population ratio to a demand change (unemployment rate of prime-age men as the instrument) by running regressions independently for seven age groups.
- Finds only old women's employment ratio is countercyclical conditional on a demand increase, not the other demographic groups.

Hornstein and Kudlyak (2019)

- Decompose the unemployment rate of 44 gender-age-education groups (from seven age groups) into long-run trend (cohort effects, age effects) and transitory cycle components.
  - those who are affected more by the cyclical factors are the ones whose unemployment rate are more volatile.
- Find the least (most) educated group's unemployment rate is the most (least) volatile, and volatility declines with age.

Limitations: Do not provide evidence on how employment of different demographic groups respond to structural shocks, such as technology shocks.

Our Approach

We explicitly model the employment to population ratio as a function of age and use a mixed autoregression (MAR).

MAR is a model for a mixture of scalar and functional variables

- VAR (vector autoregression) for scalar variables
- FAR (functional autoregression) for a functional variable

MAR allows us to

- directly analyze the dynamics of the employment curve over all ages,
- study the effects of technological progress on employment curves using the standard VAR methodologies, impulse responses, variance decomposition, and historical decomposition

We identify technology shocks via long-run restrictions on the impact of technology shocks.

Our Contribution

We provide novel empirical facts on how workers of different ages are affected by a change in technology.

We show technological progress can increase inequality between prime-age (skilled) and young and old (unskilled) workers.

Policymakers may provide more education and training to young and old (unskilled) workers, who are more vulnerable when there is a productivity increase.

Our novel empirical findings on whole employment curve can be useful for building heterogeneous-agent models with employment decisions.

- model builders may match cyclical behavior of employment of a household with young, prime-age and old members implied by their model with data

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Findings: Responses of Employment Curve to Technology Shocks

Shock	Average	Young	Prime-Age	Old	Very Old
Technology	$\downarrow^*$	$\downarrow^*$	$\downarrow^*$	$\downarrow^*$	$\downarrow^*$
	(-0.29%)	(-0.72%)	(-0.20%)	(-0.19%)	(-0.26%)

At-Impact Responses of the Employment to Population Ratio of Young, Prime-Age and Old to Technology Shocks

- Young (average of the median responses of employment at ages 16-24)
- Prime-Age (average of the median responses of employment at ages 25-54)
- Old (average of the median responses of employment at ages 55-65)
- Average (simple average of the median responses of employment at ages 16-65)

Very Old (ages 61-65)

For ages between 55-60, at-impact response is -0.13^*

$*$ signifies significance based on 90% bootstrap confidence bands

Findings: Variance Decomposition of Employment Curve

Horizons	Young (16-24)	Prime-Age (25-54)	Old (55-65)
1-quarter horizon ($h = 1$)	(37%, 63%)	(35%, 65%)	(37%, 63%)
1-year horizon ($h = 5$)	(34%, 66%)	(34%, 66%)	(37%, 63%)
3-year horizon ($h = 13$)	(34%, 66%)	(33%, 67%)	(37%, 63%)

First number (black) in parenthesis is the average contribution to forecast error variance by technology, and second number (red) by non-technology shocks.

Forecast error variance decomposition is obtained using the median of bootstrapped responses at each age.

Findings: Historical Decomposition

During the Great Recession, the decline in employment is mainly driven by non-technology shocks, not technology shocks.

The contribution to the decline in employment during the Great Recession from non-technology shocks ranges from 2 to 7 times the contribution from technology shocks depending on the age.

However, there is no clear pattern whether young, prime-age or old age group has the largest contribution from non-technology shock.

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Conventional SVAR with Long-Run Identification

As in Galí (1999), consider a structural VAR(p) model,

$$B_0 z_t = B_1 z_{t-1} + \cdots + B_p z_{t-p} + e_t,$$

where $z_t = [\triangle logPROD_t \quad logEMP_t]'$, $e_t = [e_{Tech} \quad e_{Nontech}]'$.

- Quarterly data from 1976:1 to 2019:4.
- Productivity (PROD) is calculated as real GDP over employment level.
- Employment (EMP) is employment to population ratio. Following Shimmer (2005), log (EMP) is detrended by an HP filter with a smoothing parameter of 1600.

Employment Ratio

Conventional SVAR with Long-Run Identification

As in Galí (1999), technology shock is identified as the only shock that can affect productivity in the long run. Therefore, the cumulative response of the productivity growth to non-technology shocks is 0.

For simplicity, let $p = 1$, and write the RF error ε_t in $z_t = Az_{t-1} + \varepsilon_t$ as

$$\varepsilon_t = Be_t.$$

h-period ahead and cumulative impulse responses are given by $IRF^h = A^h B$ and

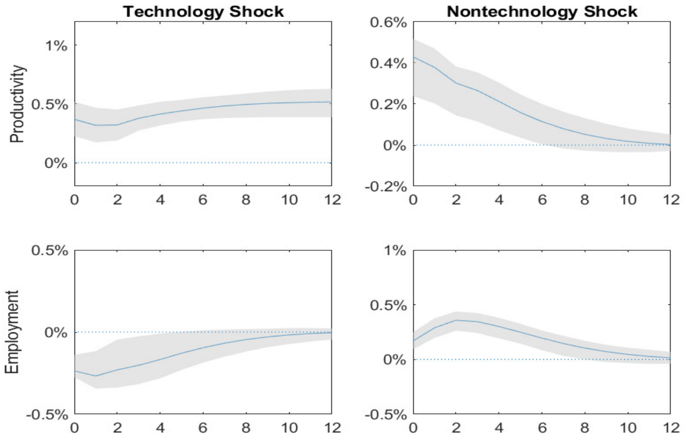
$$\theta = \sum_{h=0}^{\infty} IRF^h = (I - A)^{-1} B$$

if all eigenvalues of A have modulus less than 1. We then have

$$\theta\theta' = (I - A)^{-1} BB'(I - A)^{-1'} = (I - A)^{-1} \Sigma_\varepsilon (I - A)^{-1'}, \tag{1}$$

where A and Σ_ε are reduced form parameters that can be estimated. Under our longrun identifying restriction, θ is a lower-triangular matrix satisfying (1).

Impulse Responses from Conventional SVAR



Estimated Impulse Responses from SVAR of Galí (1999). Blue line is the point estimate and the shaded area is the 90% significance bands obtained from bootstrap.

Impulse Responses from Conventional SVAR

Response to positive technology shocks

- Employment declines by about 0.25% at impact and reaches its minimum after two quarters, and then gradually goes back to 0.
- Conditional on a technology shock, at impact, productivity and employment move in the opposite direction.

Response to positive non-technology shocks

- Employment increases by about 0.2% at impact and reaches its maximum after 3 quarters, and then gradually goes back to 0.
- Conditional on a non-technology shock, at impact, productivity and employment move in the same direction.

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Basics of Functional Analysis

Let

$$w : \ \Omega \to H$$

where H is a Hilbert space.

Hilbert-valued random variables include

- Real random variables: $H = \mathbb{R}$
- Vector-valued random variables: $H = \mathbb{R}^N$
- Function-valued random variables: $H = L^2(\mathbb{R})$

as special cases.

Mean and Variance Operator

The mean $\mathbb{E}w$ of a random variable in H is defined as a vector in H satisfying

$$\langle v, \mathbb{E}w \rangle = \mathbb{E}\langle v, w \rangle$$

for all $v \in H$, which exists if $\mathbb{E}\|w\| < \infty$.

For w such that $\mathbb{E}w = 0$, the variance $\mathbb{E}(w \otimes w)$ of w is given by an operator for which

$$\mathbb{E}\langle u, w \rangle \langle w, v \rangle = \langle u, \mathbb{E}(w \otimes w)v \rangle$$

for all $u, v \in H$, which exists if $\mathbb{E}\|w\|^2 < \infty$.

- For a finite dimensional w, $w \otimes w$ reduces to ww', and $\mathbb{E}(w \otimes w)$ reduces to $\mathbb{E}ww'$.
- For an operator A with its adjoint A^*, we may easily deduce that $\mathbb{E}(Aw \otimes Aw) = A\big[\mathbb{E}(w \otimes w)\big]A^*$.

Representation and Implementation

Let $H = L^2(\mathbb{R})$ and (w_t) be a sequence of square integrable random functions. Since $L^2(\mathbb{R})$ is separable, we may write (w_t) as

$$w_t = \sum_{i=1}^{\infty} \langle v_i, w_t \rangle v_i$$

for any orthonormal basis (v_i) of $L^2(\mathbb{R})$.

For the implementation of our subsequent methodology, we use different sets of orthonormal basis.

- In the first step, we use a Wavelet basis to establish an isomorphism between $L^2(\mathbb{R})$ and $\ell^2(\mathbb{R})$.
- In the second step, we use the functional principal component basis to interpret (v_i) as factors and $(\langle v_i, w_t \rangle)$ as factor loadings.

Define

$$H = L^2(K)$$

to be the Hilbert space of square integrable functions, and let

$$H' = \ell^2(R)$$

be the Hilbert space of square summable sequences endowed with the inner product

$$\langle z, w \rangle' = \sum_{i=1}^{\infty} z_i w_i$$

for any $w = (w_1, w_2, \ldots), z = (z_1, z_2, \ldots) \in H'$.

Let (v_i) be an orthonormal basis of H. Then we may write

$$v = \sum_{i=1}^{\infty} \langle v_i, v \rangle v_i$$

for any $v \in H$. Since

$$\langle v_i, v_j \rangle = \delta_{ij},$$

we may easily see that

$$\|v\|^2 = \sum_{i=1}^{\infty} \langle v_i, v \rangle^2,$$

which is often referred to as the Parseval's equality.

An Important Isometry

Let (v_i) be an orthonormal basis of H, and consider a mapping $\pi : H \to H'$ defined by

$$\pi(v) = \Big(\langle v_1, v\rangle, \langle v_2, v\rangle, \dots\Big).$$

Clearly, π is a bijection, i.e., it is one-to-one and onto.

Moreover, if we define $\|\cdot\|$ and $\|\cdot\|'$ to be the norms in H and H', respectively, then

$$\|v\| = \left(\sum_{i=1}^{\infty} \langle v_i, v\rangle^2\right)^{1/2} = \|\pi(v)\|'$$

due to the Parseval's equality.

This implies that π is an isometry between H and H'.

Not only the norm is defined by the inner product

$$\|v\|^2 = \langle v, v \rangle,$$

but also the inner product is defined by the norm

$$\langle u, v \rangle = \frac{1}{4}\left(\|u + v\|^2 - \|u - v\|^2\right)$$

by the parallelogram law.

Therefore, it follows immediately that

$$\langle u, v \rangle = \langle \pi(u), \pi(v) \rangle'$$

for any $u, v \in H$.

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Mixed Autoregression (MAR)

A Mixture of VAR and FAR

Our full empirical model consists of four scalar variables (productivity growth and three additional macro aggregate variables driving macro economic fluctuations and micro level employment changes) and one functional variable (detrended employment curve over ages 16-65).

Our model therefore includes both scalar and functional variables, and we let

- x_t: n-dimensional vector of scalar variables
- f_t: a functional variable

more explicitly.

Define

$$z_t = (x_t, f_t)$$

which we regard as a time series of random elements taking values in the product space $\mathcal{H} = R^n \times H$. We denote by $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$ the inner product and norm defined for H.

We endow $\mathcal{H} = R^n \times H$ with the usual inner product and norm, $\langle \cdot, \cdot \rangle_{\mathcal{H}}$ and $\| \cdot \|_{\mathcal{H}}$, for the product space, which are given by

$$\langle z, w \rangle_{\mathcal{H}} = x'y + \langle f, g \rangle \quad \text{and} \quad \|z\|_{\mathcal{H}}^2 = x'x + \|f\|^2$$

for $z = (x, f)$ and $w = (y, g)$.

Extended FAR

We let (z_t) be generated by a FAR(1) as

$$z_t = Az_{t-1} + \varepsilon_t,$$

where A is a compact linear operator in $\mathcal{H} = R^n \times H$, and (ε_t) are random elements in $\mathcal{H} = R^n \times H$ defined as

$$\varepsilon_t = (\varepsilon_t^x, \varepsilon_t^f)$$

with reduced form errors (ε_t^x) and (ε_t^f) corresponding to (x_t) and (f_t), respectively.

We may also write

$$x_t = A_{11} x_{t-1} + A_{12} f_{t-1} + \varepsilon_t^x$$
$$f_t = A_{21} x_{t-1} + A_{22} f_{t-1} + \varepsilon_t^f,$$

where $A_{11} : R^n \to R^n$, $A_{12} : H \to R^n$, $A_{21} : R^n \to H$ and $A_{22} : H \to H$ are bounded linear operators.

Without loss of generality, we may set A_{11} to be an $n \times n$ matrix, and

$$A_{21} = \left(\alpha_{21}^1, \ldots, \alpha_{21}^n\right)$$

with $\alpha_{21}^i \in H$ for $i = 1, \ldots, n$. Moreover, by the Riesz representation theorem, we may write

$$A_{12} f = \left(\langle \alpha_{12}^1, f \rangle, \ldots, \langle \alpha_{12}^n, f \rangle\right)'$$

for any $f \in H$ with $\alpha_{12}^i \in H$ for $i = 1, \ldots, n$.

Identification

We define $(n+1)$-dimensional structural shocks (e_t) as $e_t = (e_t^{x\prime}, e_t^f)'$, and let

$$\begin{pmatrix} \varepsilon_t^x \\ \varepsilon_t^f \end{pmatrix} = B \begin{pmatrix} e_t^x \\ e_t^f \end{pmatrix},$$

where

$$B : R^{n+1} \to \mathcal{H} = R^n \times H$$

is a bounded linear impact operator. For identification of the structural shocks (e_t), the operator B is specified with restrictions.

Let

$$\Sigma = \mathbb{E}(\varepsilon_t \otimes \varepsilon_t).$$

The operator B is identified if and only if there exists a unique B such that (i) B satisfies the given restrictions, and (ii) $\Sigma = BB^*$.

We let

$$\mathrm{var}\,(\varepsilon_t^f) = \mathbb{E}\big(\varepsilon_t^f \otimes \varepsilon_t^f\big),$$

and denote by (λ_i, v_i) the pairs of eigenvalues $\lambda_1 > \lambda_2 > \cdots$ and corresponding eigenvectors $v_1, v_2, \ldots$ of $\mathrm{var}\,(\varepsilon_t^f)$.

We further let

$$H_m = \mathrm{span}\,\big\{v_1, \ldots, v_m\big\},$$

and let Π_m be the projection on H_m. We also define

$$\pi_m : v \mapsto \begin{pmatrix} \langle v_1, \Pi_m v\rangle \\ \vdots \\ \langle v_m, \Pi_m v\rangle \end{pmatrix}$$

for any $v \in H$.

Clearly, π_m is an isometry between H_m and R^m.

Implementation II

We approximate B by $B_m : R^{n+1} \to \mathcal{H}_m = R^n \times H_m$ defined by

$$\begin{pmatrix} \varepsilon_t^x \\ \Pi_m(\varepsilon_t^f) \end{pmatrix} = B_m \begin{pmatrix} e_t^x \\ e_t^f \end{pmatrix}$$

Subsequently, we use the same notation to denote the $(n+m) \times (n+1)$ matrix representing B_m in the product basis given by the standard basis of R^n and $v_1, \ldots, v_m$ of H.

We have

$$\begin{pmatrix} \varepsilon_t^x \\ \pi_m(\varepsilon_t^f) \end{pmatrix} = B_m \begin{pmatrix} e_t^x \\ e_t^f \end{pmatrix}$$

In our full empirical model with four scalar variables, the four principal components of $\mathrm{var}\,(\varepsilon_t^f)$ together explain 99.59% of the total variations in (ε_t^f).

Implementation III

We write

$$\text{var}\begin{pmatrix} \varepsilon_t^x \\ \pi_m(\varepsilon_t^f) \end{pmatrix} = \sum_{i=1}^{\infty} \mu_i (w_i w_i'),$$

where (μ_i, w_i) are the pairs of eigenvalues $\mu_1 > \mu_2 > \cdots$ and corresponding eigenvectors $w_1, w_2, \ldots$ of $\text{var}\left(\varepsilon_t^{x\prime}, \pi_m(\varepsilon_t^f)'\right)'$, and define

$$\Sigma_m = \sum_{i=1}^{n+1} \mu_i (w_i w_i'),$$

which is an $(n+m)$-dimensional square matrix of rank $(n+1)$.

Now we may find B_m such that $\Sigma_m = B_m B_m'$. For B_m to be unique, we need to have $n(n+1)/2$-number of restrictions, i.e., as many number of restrictions as required to just identify SVAR consisting of $(n+1)$ variables.

▸ FPCA Implementation

Results

Conventional Structural VAR (SVAR)

- Two Scalar Variables: *PROD*, *EMP*
  - *PROD*: Growth Rate of Productivity, $\triangle logPROD_t$
  - *EMP*: Employment to Population Ratio, $logEMP_t$

Mixed Autoregression (MAR)

- One Scalar Variable: *PROD*
- One Functional Variable: *FEMP*, Employment to Population Curve over All Ages

Time Span:

- 1976.Q1 - 2019.Q4

Data

Quarterly data from 1976 Q1 to 2019 Q4 for the United States.

The by-age employment to population ratio is calculated by using monthly CPS data from Integrated Public Use Microdata Series (IPUMS) and then deseasonalized by X-13 ARIMA-SEATS. The quarterly data is obtained by taking quarterly averages of monthly data.

Data Sources:

- Real GDP: taken from the FRED. Quarterly, Billions of Chained 2012 Dollars, Seasonally Adjusted, GDPC1.
- Employment level: taken from the FRED. Quarterly, Thousands of Persons, Seasonally Adjusted, CE16OV.
- By-Age Employment to Population Ratio: IPUMS-CPS, University of Minnesota, www.ipums.org.

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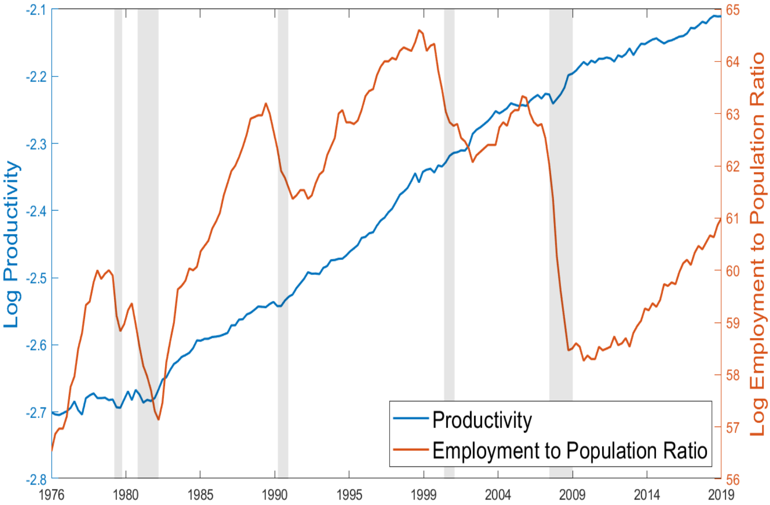
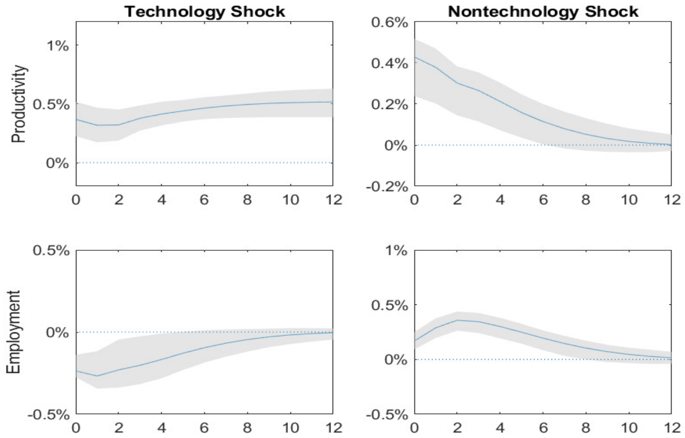
SVAR with Aggregate Employment

Aggregate Productivity and Aggregate Employment



Impulse Responses from Conventional SVAR



Estimated Impulse Responses from SVAR of Galí (1999). Blue line is the point estimate and the shaded area is the 90% significance bands obtained from bootstrap.

Impulse Responses from Conventional SVAR

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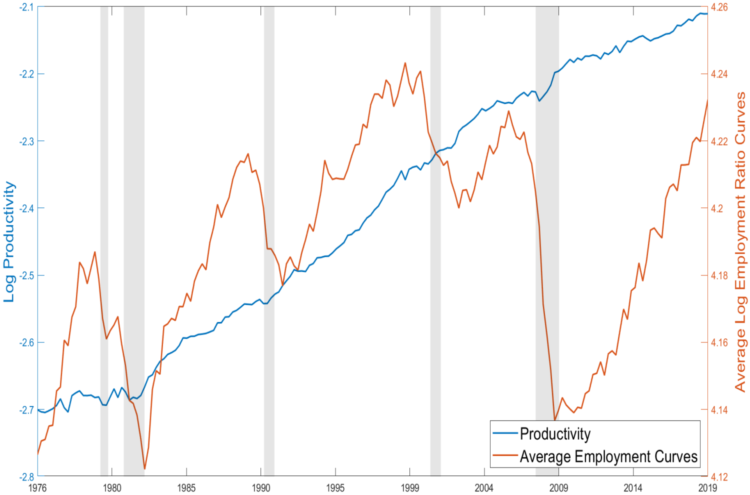
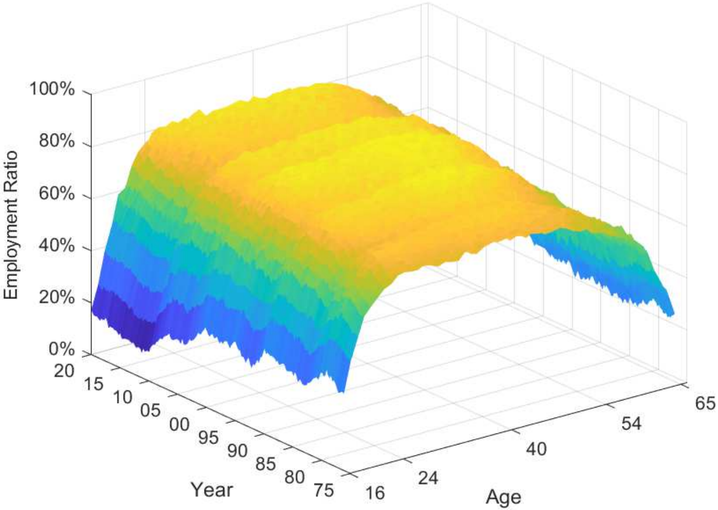
MAR with Employment Curve

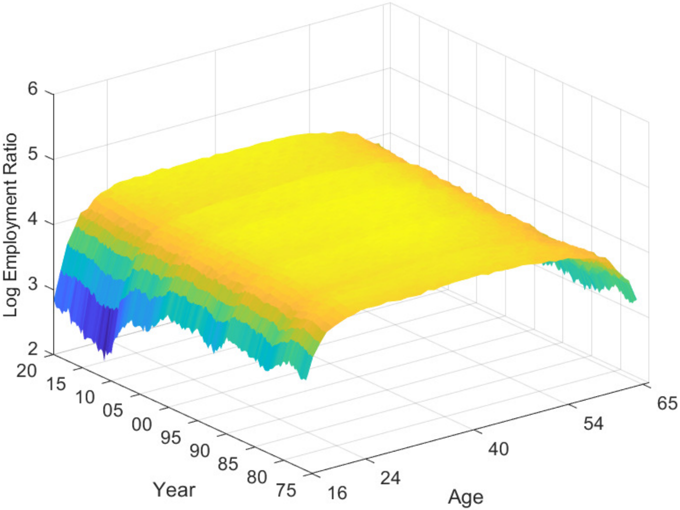
Aggregate Productivity and Average of Employment Curves

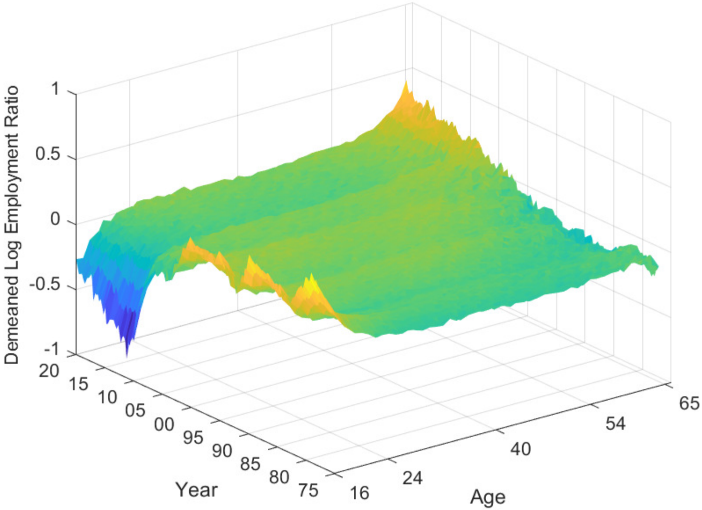


Employment Curves

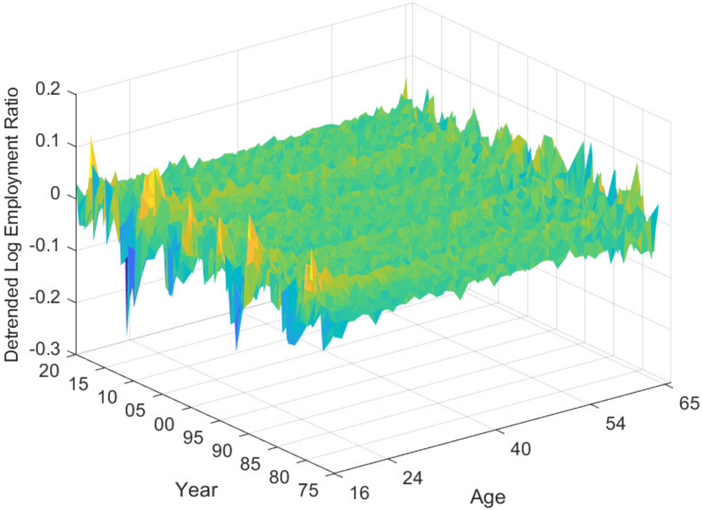


Employment Curves (in logs)

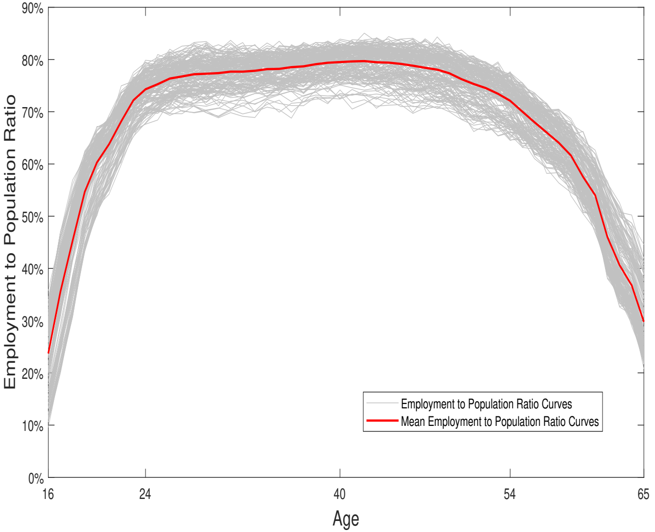



Demeaned Log Employment Ratio
1
0.5
0
-0.5
-1
20
15
10
05
00
95
90
85
80
75
Year
16
24
40
54
65
Age

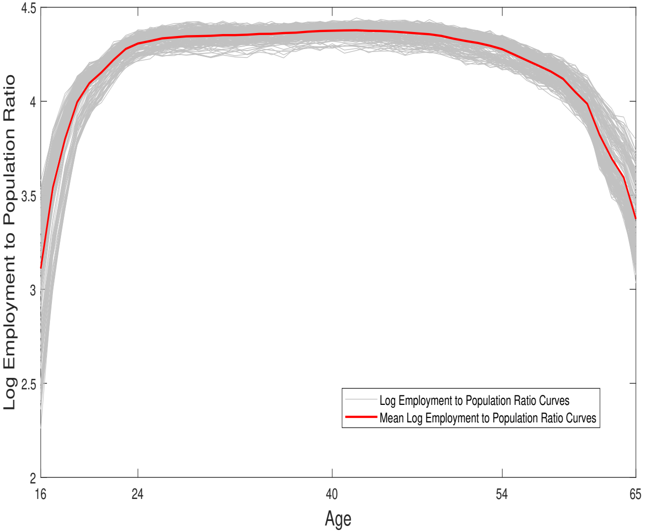
Employment Curves (Detrended)



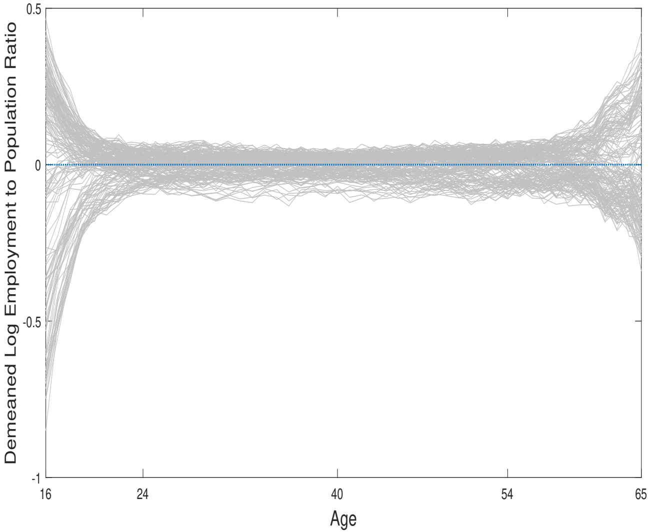
Employment Curves - 2D



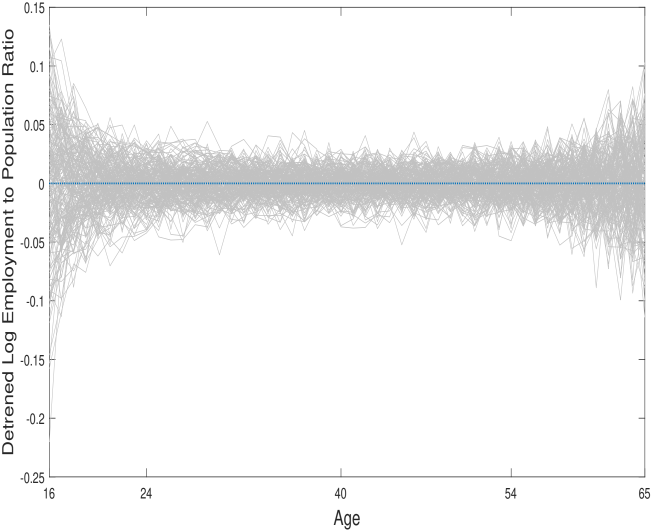
Employment Curves (in logs) - 2D



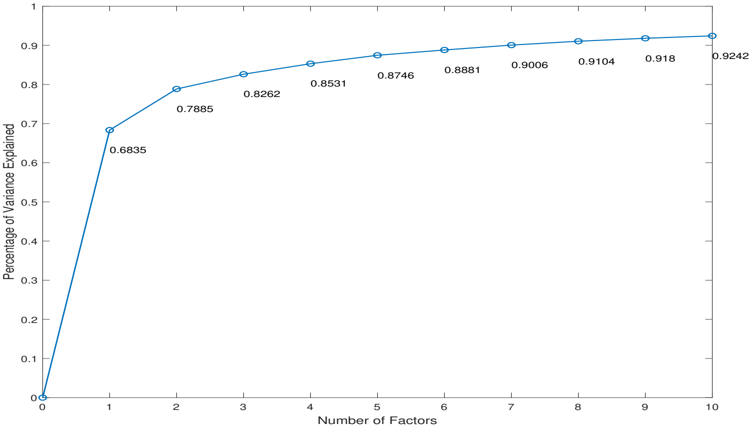
Employment Curves (Demeaned) - 2D



Employment Curves (Detrended) - 2D

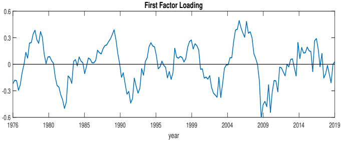


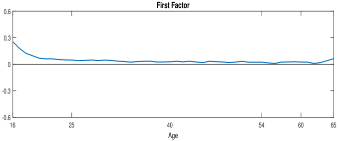
Cumulative Scree Plot

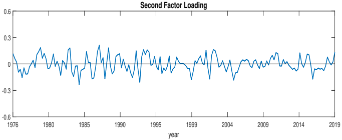


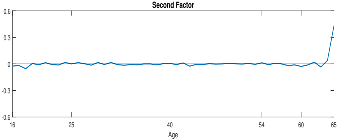
Four leading factors explain over 85% of the variations in the time series of employment curves.

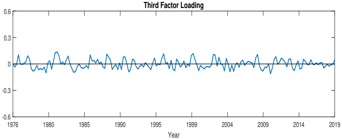
Factors and Factor Loadings

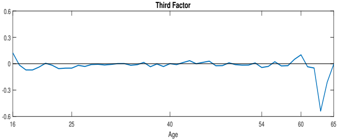


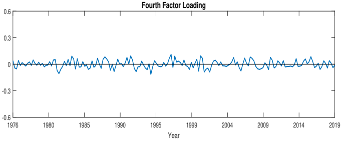


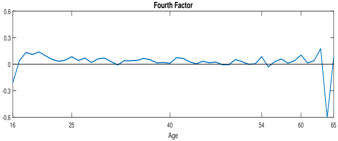




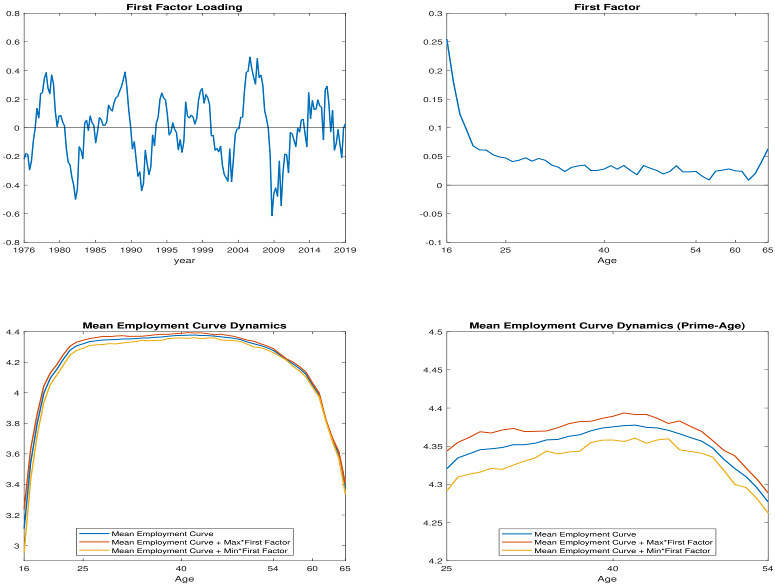




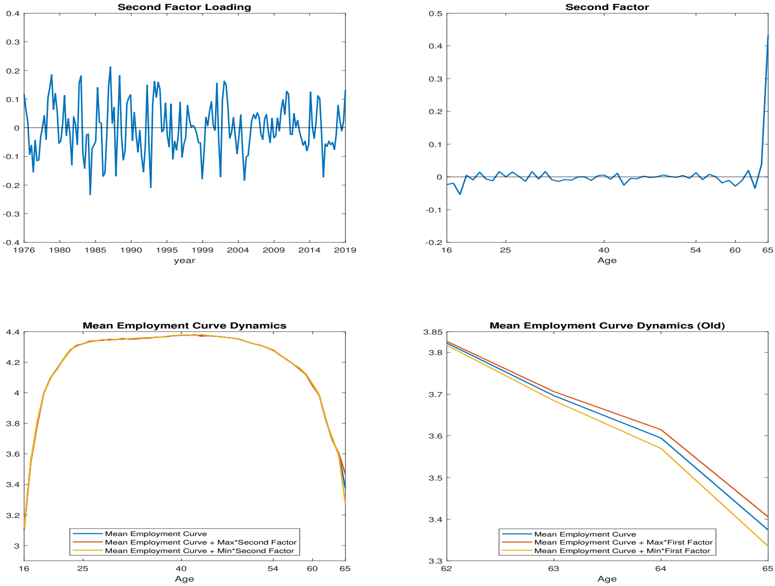




First Factor



Second Factor



Extend Galí (1999) to MAR

We let $(z_t) = (x_t, f_t)$ be generated as

$$z_t = Az_{t-1} + \varepsilon_t,$$

where A is a compact linear operator in $\mathcal{H} = R \times H$, and (ε_t) are random elements in $\mathcal{H} = R \times H$ defined as

$$\varepsilon_t = (\varepsilon_t^x, \varepsilon_t^f)$$

with reduced form errors (ε_t^x) and (ε_t^f) corresponding to

- (x_t): growth of log productivity
- (f_t): employment curve with cross-sectional employment ratios across ages 16-65.

We define 2-dimensional structural shocks (e_t) as $e_t = (e_t^{x\prime}, e_t^f)'$, where e_t^x is the technology shock, and e_t^f is the non-technology shock, and let

$$\begin{pmatrix} \varepsilon_t^x \\ \varepsilon_t^f \end{pmatrix} = B \begin{pmatrix} e_t^x \\ e_t^f \end{pmatrix},$$

where $B : R^2 \to \mathcal{H} = R \times H$ is a bounded linear impact operator

For identification of structural shocks $e_t = (e_t^x, e_t^f)$, B is specified with a restriction: technology shock is the only shock that affects productivity in the long run.

Long-Run Identification

Let

$$\theta = \begin{bmatrix} \theta_U \\ \theta_L \end{bmatrix},$$

where the upper part of θ, θ_U is $(n+1) \times (n+1)$ dimensional lower-triangular square matrix, and the lower part of θ, θ_L is $(m-1) \times (n+1)$ dimensional matrix.

For long run identification we solve

$$\min_{\theta} \|\theta\theta' - (I-A)^{-1}\Sigma_m(I-A)^{-1'}\|^2$$

s.t. θ_U is a lower-triangular matrix.

We solve this using QR Decomposition:

- Let $(I-A)^{-1}\Sigma_m(I-A)^{-1'} = V\Lambda V'$, which is the singular value decomposition of the matrix.
- Next, compute the QR decomposition of $\Lambda^{\frac{1}{2}} V' = QR$, where Q is an orthogonal matrix and R is an right triangular matrix. Then we have $(I-A)^{-1}\Sigma_m(I-A)^{-1'} = V\Lambda^{\frac{1}{2}}\Lambda^{\frac{1}{2}} V' = R'Q'QR = R'R$.
- This will give us $\theta = R'_{n+1}$, where R_{n+1} is the first $n+1$ rows of R.

Since $n = 1$ and $m = 4$ in our simple model, we have

$$\Lambda^{\frac{1}{2}} V' = QR = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 \end{bmatrix} \times \begin{bmatrix} r_1' \\ r_2' \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where the last three rows of R is zero due to the fact that Σ_m is reduced ranked, and the rank of Σ_m is $n + 1 = 2$.

Thus we have

$$(I - A)^{-1} \Sigma_m (I - A)^{-1'} = R'R = \begin{bmatrix} r_1 & r_2 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} r_1' \\ r_2' \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 \end{bmatrix} \begin{bmatrix} r_1' \\ r_2' \end{bmatrix}$$

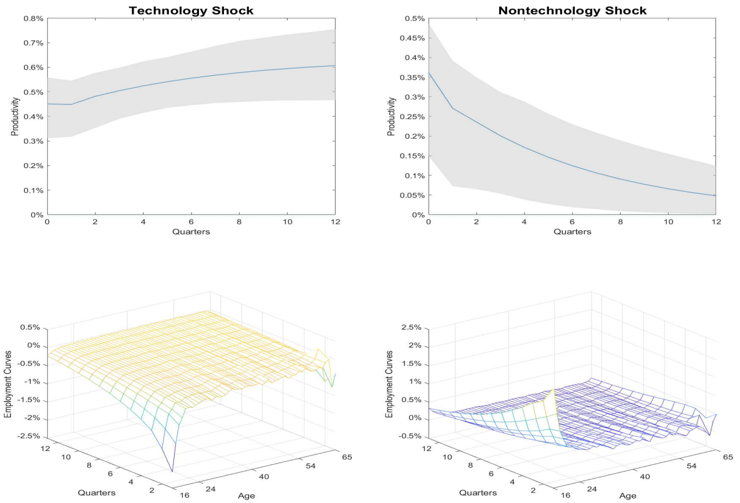
which, together with

$$(I - A)^{-1} \Sigma_m (I - A)^{-1'} = R'_{n+1} R_{n+1},$$

identifies the longrun impact matrix θ as

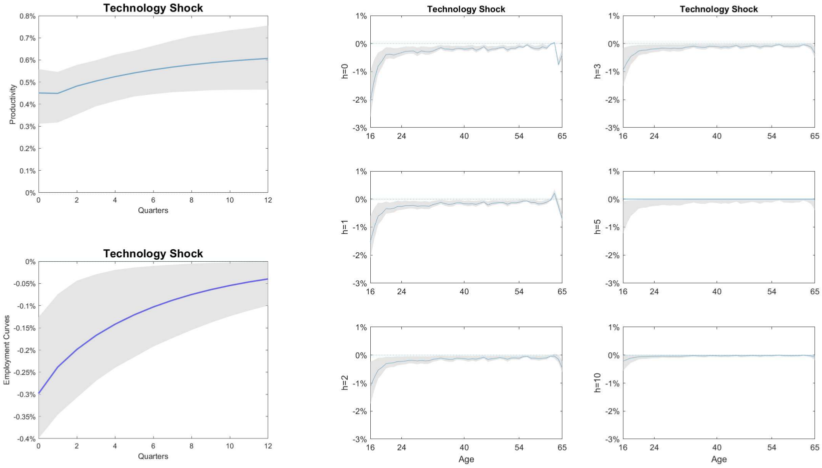
$$\theta = R'_{n+1}$$

MAR Impulse Responses to Technology Shock

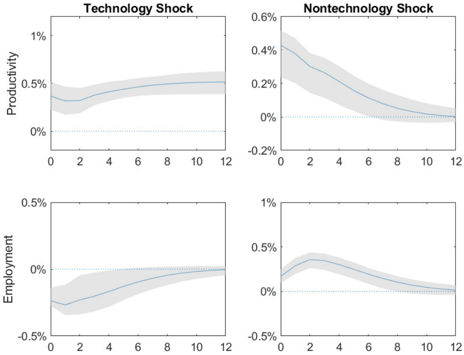


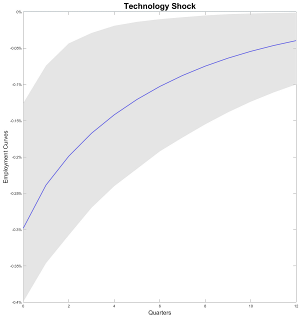
Upper (Lower): Responses of Productivity Growth (Employment Curve)

MAR Impulse Responses to Technology Shock - 2D Slices



Left: Upper (Lower): Responses of Productivity (Average Responses of Employment Curves). Right: Responses of Employment Curves.





Left: Results from Conventional SVAR of Galí (1999); Right: Results from MAR.

Impulse Responses to Technology Shock - Discussion

Heterogeneous Responses of the Employment Curves:

- At impact, employment of all ages responds negatively to a positive technology shock. In terms of magnitude young$>$old$>$ prime-age.
- The decline in young's employment can be at most three times the decline in old's employment.
- Our results show the negative response of aggregate/average employment to technology shocks mainly comes from the young.

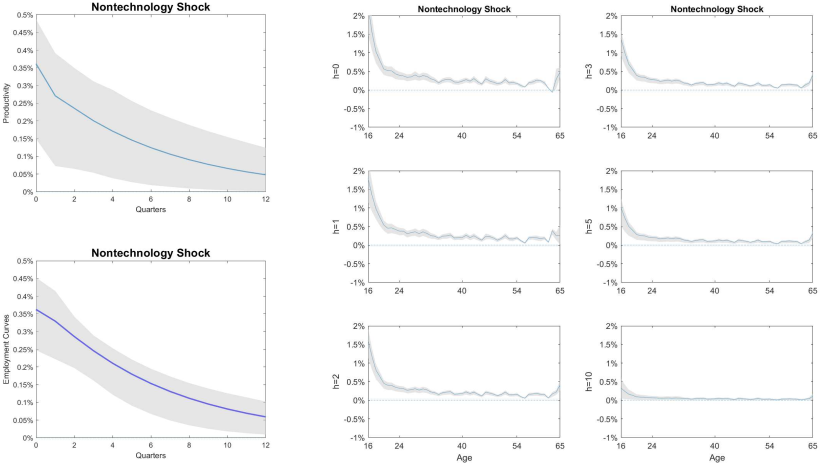
Average Responses of the Employment Curves:

- On average, employment declines by about 0.3% after a positive technology shock, then gradually goes back to 0, which is consistent with the aggregate results in Galí (1999).

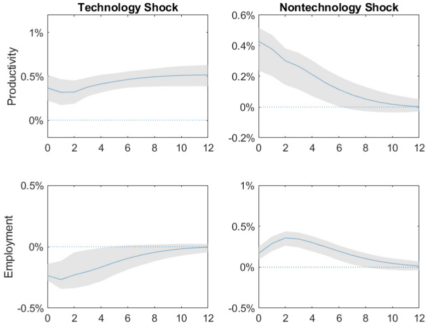
Impulse Responses to Technology Shock - Implications

- Galí (1999) pointed out sticky prices could lead to the negative response of employment to technology shocks. When the prices are sticky, firms want to reduce the price but they can't. Instead, firms will produce output to meet the demand with less labor input with more advanced technology.
- Our results show with sticky prices, the less demand for labor introduced by a technology shock affects the young and the old more, which is consistent with capital-skill complementarity in production (Krusell et. al., 2000).
- Advanced technology can increase the inequality between skilled and unskilled workers.
- Policymakers should provide more education and training to the young and the old, who are unskilled workers.

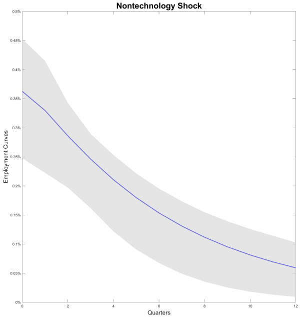
MAR Impulse Responses to Non-technology Shock



Left Upper(Lower): Responses of Productivity (Average Responses of Employment Curves). Right: Response of Employment Curves.



(a) Galí (1999) Replication Results



(b) Result from MAR model

Left: Results from Conventional SVAR of Galí (1999); Right: Results from MAR

Impulse Responses to Non-technology Shock - Discussion

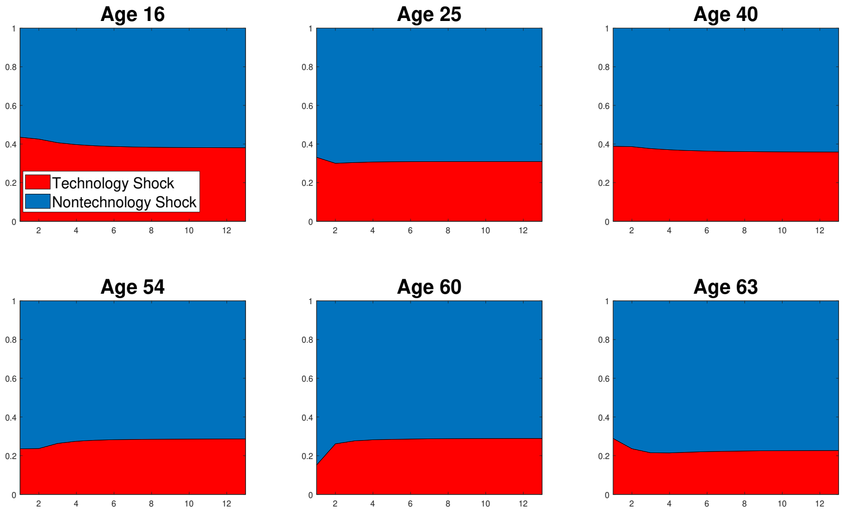
Results:

- On average, employment responds to non-technology shocks positively at impact. It increases by about 0.2% after a non-technology shock. Our results are thus consistent with the aggregate results of Galí (1999).
- On average, productivity and employment move in the same direction, conditional on a positive technology shock.
- At impact, employment of all ages respond positively, and the young's response is the largest. Our results show the positive response of aggregate/average employment to non-technology shocks mainly comes from the young.

Implications

- A non-technology shock (such as demand shocks, monetary and fiscal policy shocks) may help improve the employment of the young more significantly than other ages. This provides an empirical support for a public policy to promote employment of the young.

MAR Forecast Error Variance Decomposition



Variance Decomposition of the functional variable (employment curve) at six specific ages: 16 (young), 25, 40, 54 (prime-age), 60, 63 (old)

MAR Variance Decomposition by Ages - Discussion

Results:

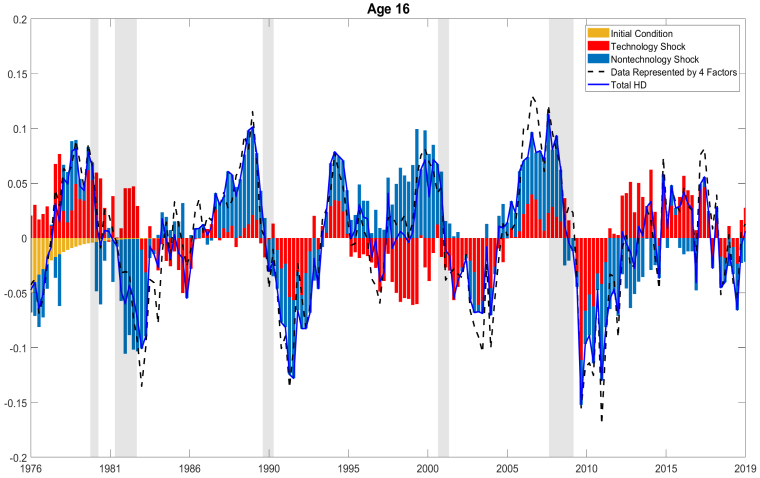
- Technology shocks explains about 30%-40%, and non-technology shocks explains around 60% the variations in the young and the prime-age's employment in all horizons, and non-technology shocks become more important as age increase.
- For the variations in old's employment, non-technology shocks explains about $60\% - 80\%$ in all horizons for most ages (except age 55 and 64).

Implications:

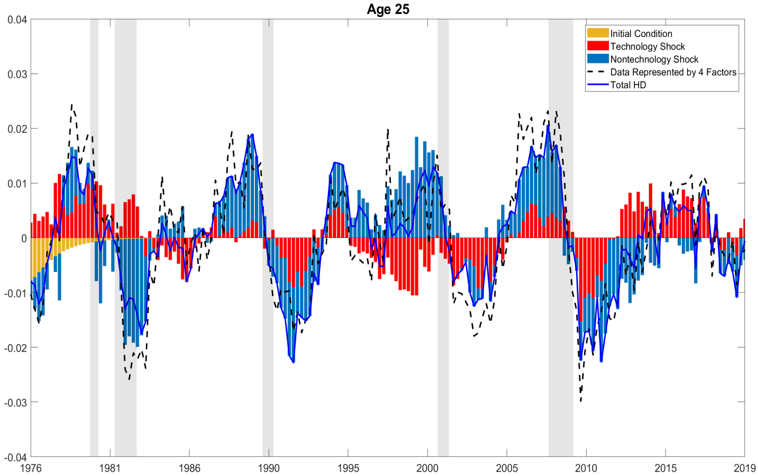
- Both demand and supply factors are relatively important in driving the employment fluctuations of young and prime-age workers.
- However, as age increases, labor supply factors (due to non-technology shocks) seems to become more important in driving the employment fluctuations.

Next six slides presents HD (Historical Decomposition) of employment curve at six specific ages: 16 (young), 25, 40, 54 (prime-age), 60, 63 (old)

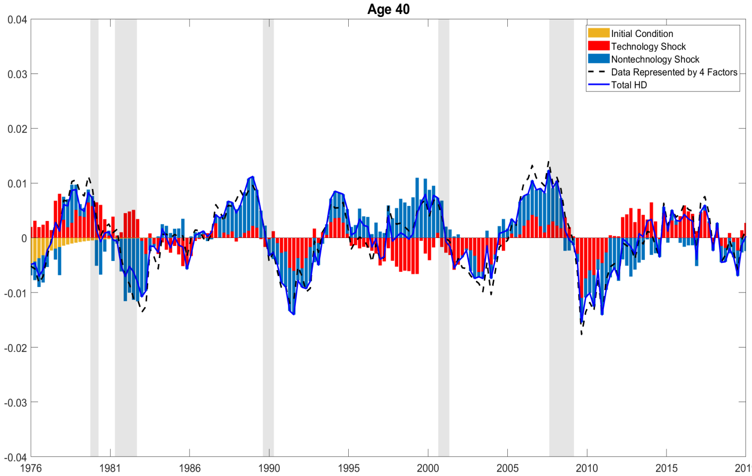
MAR Historical Decomposition of Age 16



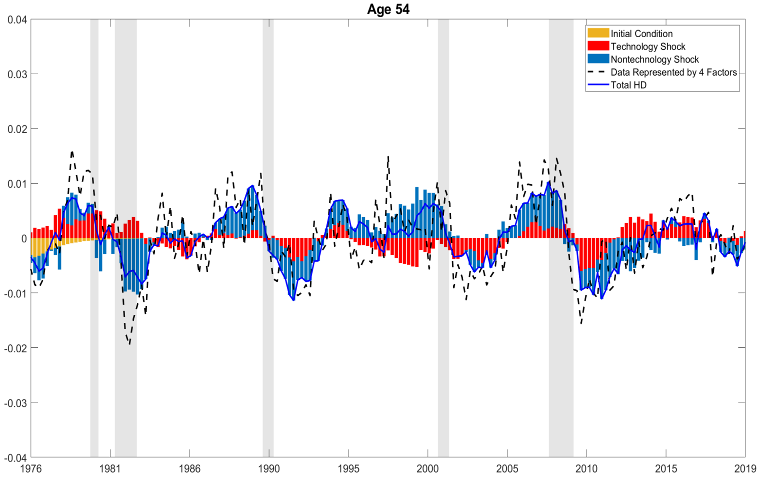
MAR Historical Decomposition of Age 25



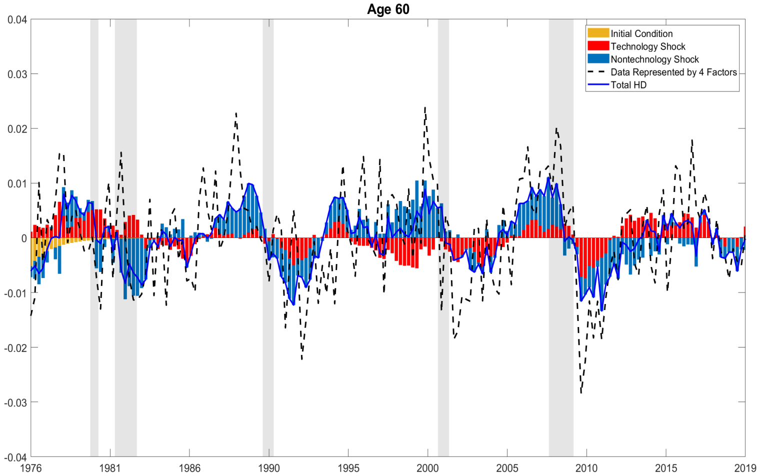
MAR Historical Decomposition of Age 40



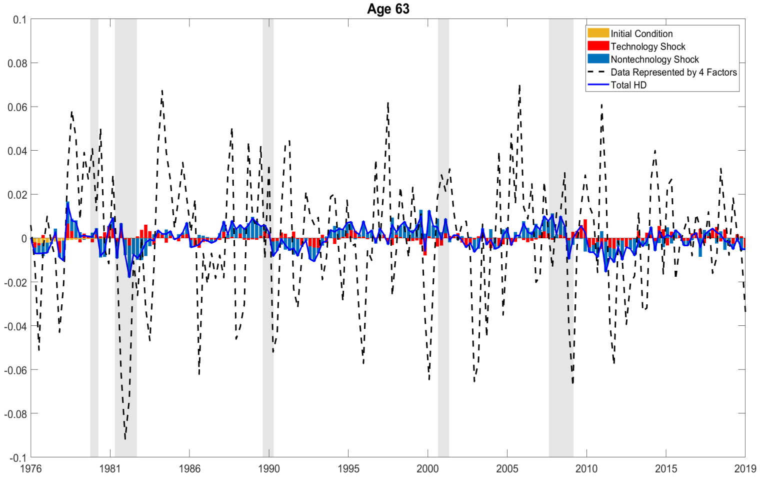
MAR Historical Decomposition of Age 54



MAR Historical Decomposition of Age 60



MAR Historical Decomposition of Age 63



Conclusions

- In a mixed autoregression (MAR), we show the employment of ages 16-65 does respond heterogeneously to technology shocks.
- We found at impact, employment of all ages responds negatively to a positive technology shock. In terms of magnitude young>old> prime-age. The decline in young's employment can be at most three times the decline in old's employment. The results imply a positive technology shocks can increase inequality between skilled and unskilled workers.
- We also found technology shocks explains about 30%-40%, and non-technology shocks explains about 60% the variations in the young and the prime-age's employment in all horizons, and non-technology shocks become more important as age increase. For the variations in old's employment, non-technology shocks is explaining about $60\% - 80\%$ in all horizons for most ages (except age 55 and 64).

MAR Historical Decomposition - Discussion

- Decline in employment aftermath of the recent Great Recession, Global Financial Crisis (GFC), is mainly driven by non-technology shocks, not technology shocks.
- Depending on aga, the contributions from non-technology shocks to the decline in employment during this period can ranging from 2 to 7 times those from technology shocks.
- However, there is no clear evidence showing which age group, young, prime-age or old, is most affected by non-technology shock.

- Black-dotted line is our detrended data represented by four functional factors. The blue line is the sum of the HD due to initial condition, technology shocks, and non-technology shocks (sum of the yellow, red and blue bars).
- Observed discrepancy between the black-dotted and blue lines is due to our identification scheme which reduces the rank of RE error variance Σ from $n+m$ to $n+1$ (from 5 to 2).
- Discrepancy is substantial and much larger for the old, compared to those of the young and prime-age.
- Discrepancy at older ages, especially the oldest group 60-65, persist even when we increase the number of factors, suggesting additional macro aggregate variables are needed to capture employment dynamics at to these old ages.
- We apply adaptive Lasso to these discrepancies at ages 60-65 with FRED-QD, and select three additional macro variables to augment our baseline model. Details follow.

Extended MAR: Impulse Responses to Technology Shocks

Impact impulse responses of the employment of the young, prime-age and old to technology shocks from the extended MAR with additional variables selected from adaptive LASSO.

Shock	Average	Young	Prime-Age	Old	Very Old (61-65)
Technology	$\downarrow^*$	$\downarrow^*$	$\downarrow^*$	$\downarrow$	$\downarrow$
	(-0.29%)	(-0.71%)	(-0.21%)	(-0.18%)	(-0.24%)

- Young (average of the median responses of employment at ages 16-24)
- Prime-Age (average of the median responses of employment at ages 25-54)
- Old (average of the median responses of employment at ages 55-65)
- Average (simple average of the median responses of employment at ages 16-65)

$*$ signifies significance based on 90% bootstrap confidence bands. It is -0.13^* for ages between 55-60.

Extended MAR: Variance Decomposition

Variance Decomposition (VD) of the employment curves from extended MAR with additional variables selected from adaptive Lasso.

Horizons	Young (16-24)	Prime-Age (25-54)	Old (55-65)
1-quarter horizon ($h = 1$)	$(36\%, 64\%)$	$(33\%, 67\%)$	$(14\%, 86\%)$
1-year horizon ($h = 5$)	$(36\%, 64\%)$	$(34\%, 66\%)$	$(18\%, 82\%)$
3-year horizon ($h = 13$)	$(35\%, 65\%)$	$(34\%, 66\%)$	$(18\%, 82\%)$

- Numbers in the parenthesis are the average variance decomposition of the employment due to technology shock (black) and non-technology shocks (red)
- VD is computed using the median of the bootstrapped responses.
- Contribution from non-technology shocks is the sum of the contributions from the shocks corresponding to the second, third, fourth aggregate variables and the functional variable.

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   - Results
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Variables Selected by Adaptive LASSO

Difference between red and green	Variables Selected from FRED-QD [Coefficient]
Age 60	SLCEx [0.0252]
Age 61	PERMITNE[0.1470], DGOERG3Q086SBEA [0.0633] UEMPMEAN [0.0393], PRFIx [-0.0294] RSAFSx [-0.1115], NWPIx [-0.1658] B021RE1Q156NBEA [-0.2012]
Age 62	RSAFSx [-0.0275]
Age 63	DGOERG3Q086SBEA [0.1345]
Age 64	SLCEx [-0.0721]
Age 65	SLCEx [-0.0356]
Ages 60-65 (average)	PERMITNE[0.1778], BAA[0.1245] GS1 [0.0321],NWPIx [-0.05] B021RE1Q156NBEA [-0.1857], RSAFSx [-0.2589]

Variables Selected by Adaptive LASSO - Details

Category	Variables (transformation)	Details
Prices	DGOERG3Q086SBEA ($\triangle^2 log(x_t)$)	Personal consumption expenditures: Nondurable goods: Gasoline and other energy goods (chain-type price index)
Inventories, Orders and Sales	RSAFSx ($\triangle log(x_t)$)	Real Retail and Food Services Sales (Millions of Chained 2012 Dollars), deflated by Core PCE
NIPA	SLCEx ($\triangle log(x_t)$)	Real government state and local consumption expenditures (Billions of Chained 2012 Dollars), deflated using PCE
	B021RE1Q156NBEA ($\triangle x_t$)	Shares of gross domestic product: Imports of goods and services (Percent)
	PRFIx ($\triangle log(x_t)$)	Real private fixed investment: Residential (Billions of Chained 2012 Dollars), deflated using PCE
Housing	PERMITNE ($\triangle log(x_t)$)	New Private Housing Units Authorized by Building Permits in the Northeast Census Region (Thousands, SAAR)
Interest Rate	BAA ($\triangle x_t$)	Moody's Seasoned Baa Corporate Bond Yield© (Percent)
	GS1 ($\triangle x_t$)	1-Year Treasury Constant Maturity Rate (Percent)
Non-Household Balance Sheets	NWPIx (x_t)	Net Worth of Households and Nonprofit Organizations Relative to Disposable Personal Income (Percent)
Employment and Unemployment	UEMPMEAN($\triangle x_t$))	Average (Mean) Duration of Unemployment (Weeks)

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Summary of Lasso Results

- We added three variables, 'SLCEx','RSAFSx', and 'PRFIx' (transformation: $\triangle log(x_t)$) selected from Lasso regression as as the second, third,$\cdots$, aggregate variables in FSVAR (first aggregate variable is productivity growth).
- Adding the three variable significantly improved the discrepancy between the data represented by 4 factors and the data explained by our FSVAR model (which can be calculated as the sum of HDs) in the baseline model.
- Comparing to the IRFs in the baseline model, after adding the three variables selected from Lasso, the on-impact responses of ages 56,60 and 62-65 are not significant with 90% confidence band, though the median response is negative.

Aggregate Data and Average of Employment Curves

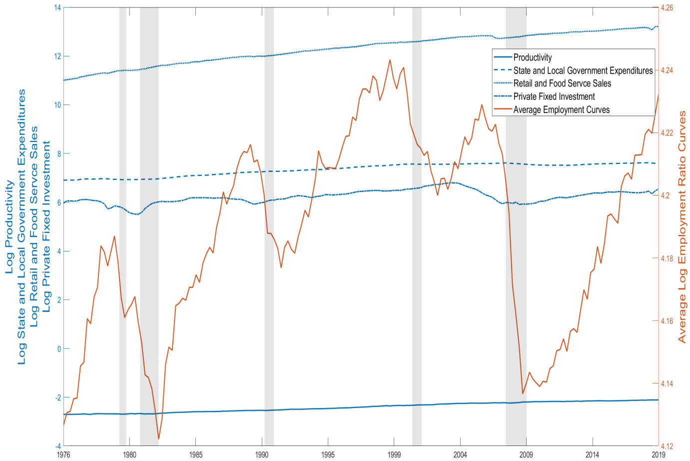


Figure: Aggregate Data and Average of Employment Curves (in logs)

Correlation Matrix of Lasso Variables with Productivity

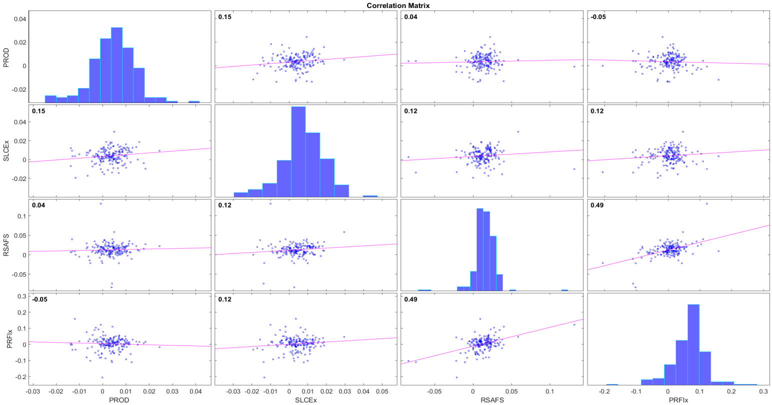


Figure: Correlation Matrix of Lasso Variables

The variables have little correlations, thus adding variables selected from selecting from Lasso is adding new information to our analysis.

IRFs of Aggregate Variables and Employment Curves to Technology Shocks

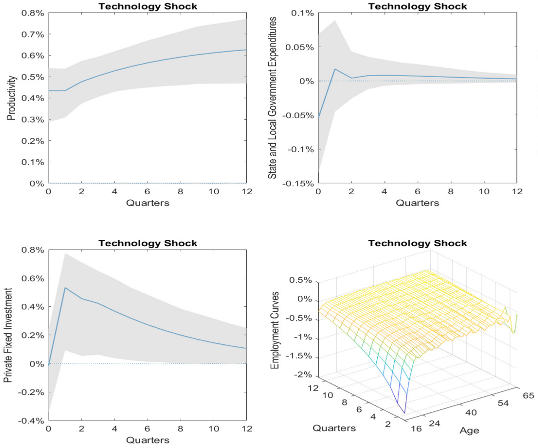


Figure: mpulse Responses of Aggregate Variables and Employment Curves to Technology Shocks from the Functional SVAR Model with Lasso Variables.The blue line is the point estimate and the shaded area is the 90% significance bands obtained from bootstrap.

MAR Impulse Responses to Technology Shock - 2D Slices

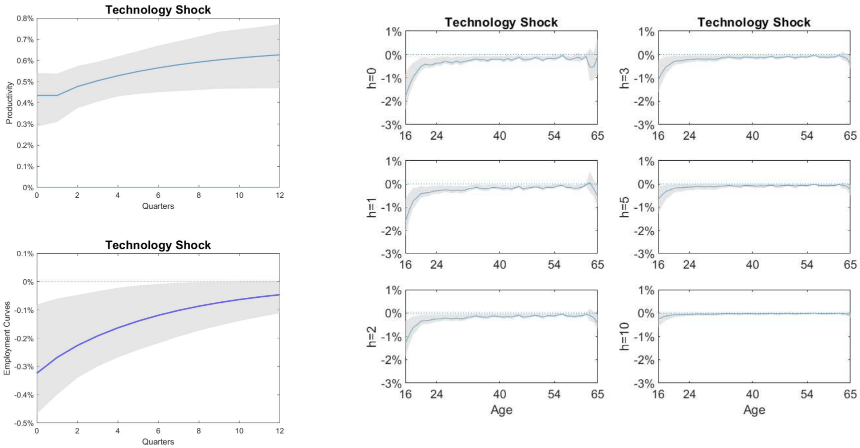
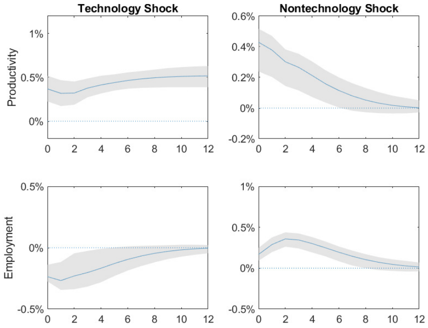


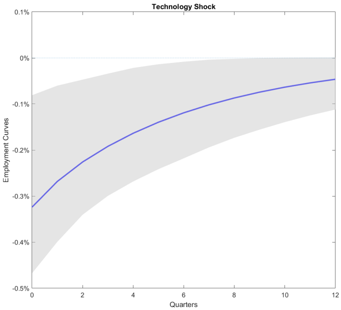
Figure: Upper Left: Average Responses of Employment Curves. Lower Left: Responses of Productivity. Right: Response of Employment Curves. All responses come from MAR model with added variables selected from Lasso. The blue line is the point estimates and the shaded area is the 90% significance bands from bootstrap.

Aggregate/Average Responses - SVAR vs. MAR

- Our results confirm the aggregate results of Galí (1999): On average, employment curves respond to technology shocks negatively at impact.



(a) Galí (1999) Replication Results



(b) Result from MAR

Figure: Compare Aggregate Results from Galí (1999) and Average Responses of Employment Curves in MAR model with added variables selected from Lasso

Impulse Responses to Technology Shock - Discussion

Heterogeneous Responses of the Employment Curves:

- Similar to the baseline model, we found at impact, employment of all ages responds negatively to a positive technology shock. In terms of magnitude young$>$old$>$ prime-age.
- Our results show the negative response of aggregate/average employment to technology shocks mainly comes from the young.
- In the baseline model, the negative on-impact responses of employment across almost all ages are significant within a 90% confidence band, except age 63. However, the responses are not significant for ages 56,60, and 62-65 within a 90% confidence band in the model with Lasso variables.

Average Responses of the Employment Curves:

- On average, employment declines by about 0.3% after a positive technology shock, then gradually goes back to 0, which is consistent with the aggregate results in Galí (1999).

Extended MAR: Forecast Error Variance Decomposition

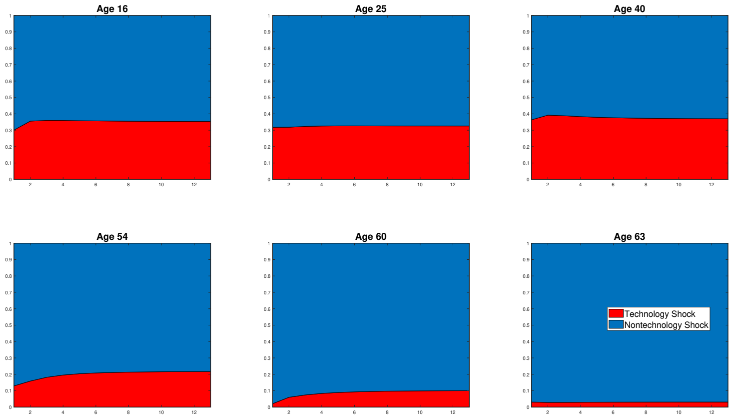


Figure: Variance Decomposition by ages in the MAR model. (Shocks corresponding to the lasso variables and functional variables are combined as non-technology shocks.)

Extended MAR: Variance Decomposition - Discussion

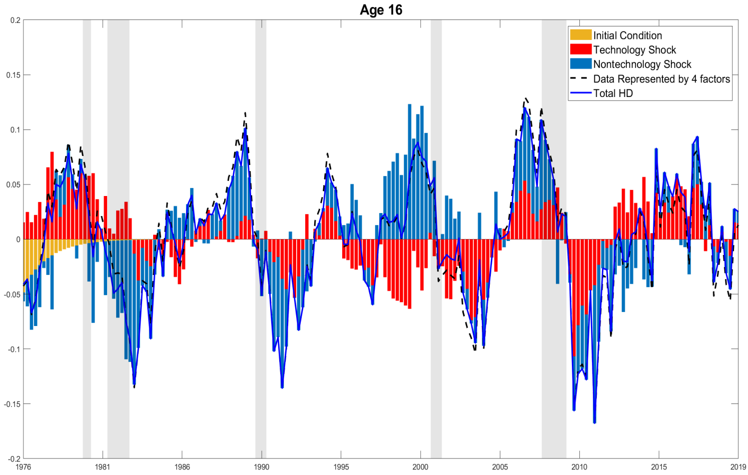
Results:Similar to baseline model, we also found technology shocks explains about 30%-40%, and non-technology shocks explains more than 60% the variations in the young and the prime-age's employment in all horizons, and non-technology shocks become more important as age increase.Compared to baseline model, the contribution from nontechnology shocks is larger for the old in the model with Lasso variables, in which non-technology shocks is explaining more than 80% variations in old's employment in all horizons (except ages 55, 57, 58, 61, which are $60 - 70\%$). In the baseline model, it is about $60\% - 80\%$ in all horizons for most ages (except age 55 and 64).

Implication:

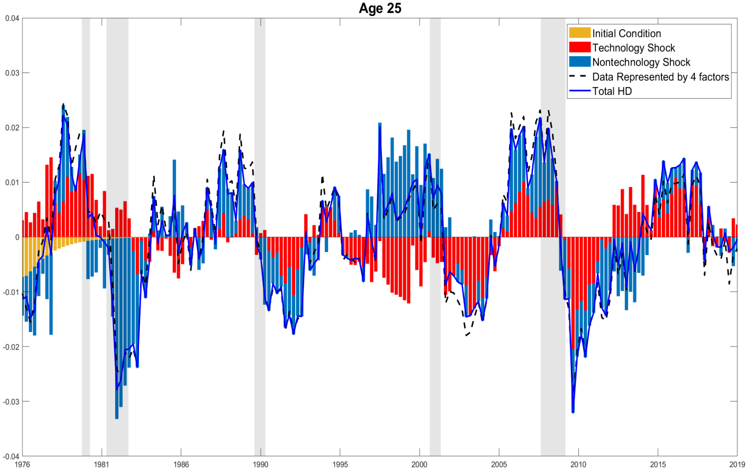
- Both demand and supply factors are relatively important in driving the employment fluctuations of young and prime-age workers. However, as age increase, labor supply (due to non-technology shocks) are becoming more important in driving the employment fluctuations.

Next six slides presents HD (Historical Decomposition) of employment curve at six specific ages: 16 (young), 25, 40, 54 (prime-age), 60, 63 (old)

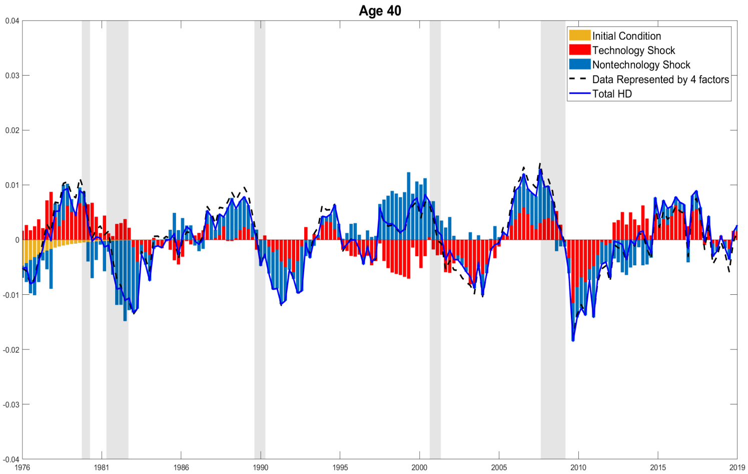
Extended MAR: Historical Decomposition of Age 16



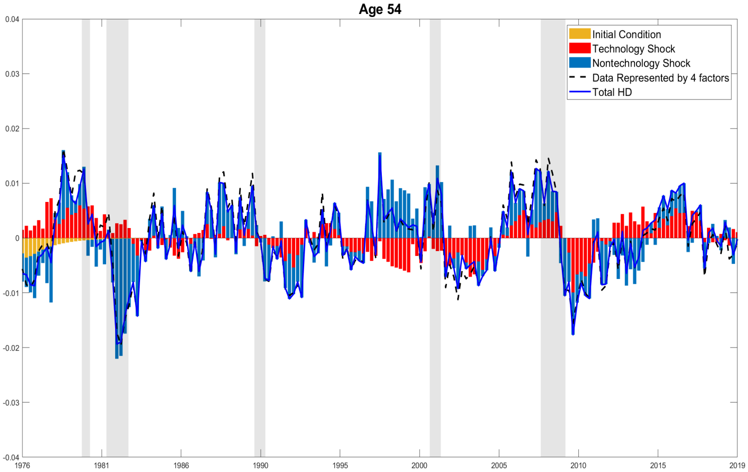
Extended MAR: Historical Decomposition - Age 25



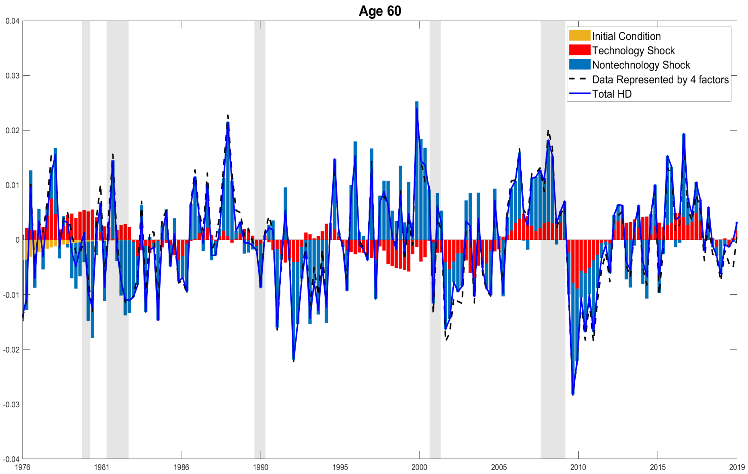
Extended MAR: Historical Decomposition - Age 40



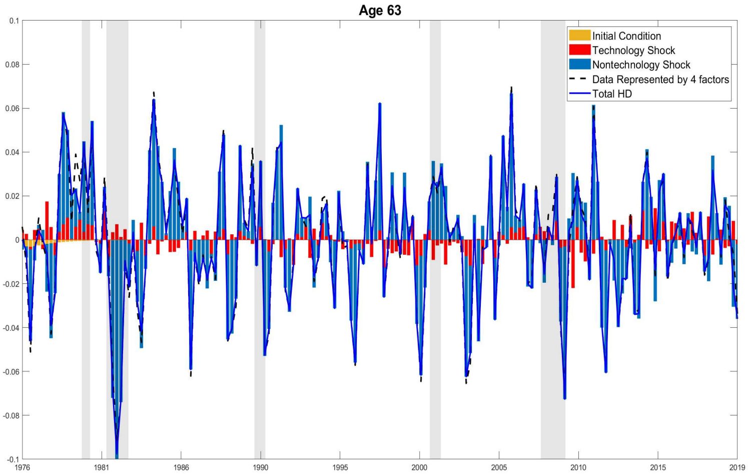
Extended MAR: Historical Decomposition - Age 54



Extended MAR: Historical Decomposition - Age 60



Extended MAR: Historical Decomposition - Age 63



MAR with Aggregate Variables from Galí (1999) 5-Variable VAR

Aggregate Variables:

- The specification considered uses data on money, interest rates, and prices, in addition to the productivity and labor-input series used in the bivariate model.
- Stock of money is the (log) of M2. The price measure is the (log) of the consumer price index (CPI) . The nominal interest rate is the three-month Treasury Bill rate.

Shocks:

- Technology and non-technology shocks (4 shocks together)

Data:

- **Real GDP**: obtained from FRED website, series GDPC1, Billions of Chained 2012 Dollars, Seasonally Adjusted.
- **Real M2 Money Stock**: obtained from FRED website, series M2REAL, Billions of 1982-84 Dollars, Seasonally Adjusted.
- **CPI**: Consumer Price Index for All Urban Consumers: All Items in U.S. City Average, obtained from FRED website, series CPIAUCSL, Index $1982 - 1984 = 100$, Seasonally Adjusted.
- **3-Month Treasury Bill: Secondary Market Rate**: obtained from FRED website, series DTB3, Percent, not seasonally adjusted.

Compare Historical Decomposition of Age 63

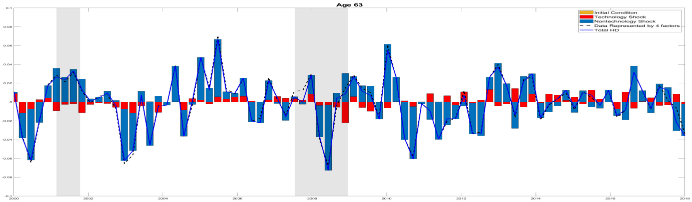


Figure: MAR model with additional aggregate variables selected by adaptive Lasso (Real Retail and Food Services Sales, Real government state and local consumption expenditures, Real private fixed investment: Residential)

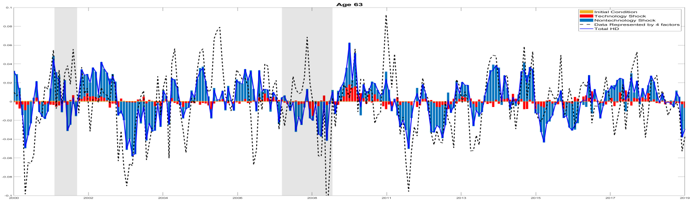


Figure: MAR model with additional aggregate Variables from Galí (1999) 5-Variable VAR (Real M2, CPI, 3-Month Treasury Bill Rate)

Conclusions (based on Results from Extended MAR)

- In a mixed autoregression (MAR), we show the employment of ages 16-65 does respond heterogeneously to technology shocks.
- We found at impact, employment of all ages responds negatively to a positive technology shock. In terms of magnitude young>old> prime-age. However, the responses are not significant for ages 56,60, and 62-65 within a 90% confidence band in the model with Lasso variables.
- We also found technology shocks explains about 30%-40%, and non-technology shocks explains about 60% the variations in the young and the prime-age's employment in all horizons, and non-technology shocks become more important as age increase. For the variations in old's employment, non-technology shocks is explaining more than 80% in all horizons (except age 55, 57, 58, 61, which are $60-70\%$).

Appendix

Literature on Aggregate Employment

Economics Question: How does employment respond to important structural shocks, such as technology shocks in business cycles?

- In a RBC model, employment/hours will rise after a technology shock, and decline after a non-technology shock.
- Mechanism: Technology shocks shift the labor demand schedule right, and non-technology shocks shift the labor supply schedule left.

Galí (1999)

- The paper finds employment/hours responds negatively to a technology shock, and positively to a non-technology shock in a structural VAR.[1]
- These empirical evidence is more consistent with a model with imperfect competition, sticky prices, and variable efforts, not RBC models.
- Mechanism: When prices are sticky, firms want to reduce prices but they can't. Instead, to meet the demand, firms will produce with less labor input due to increased productivity.

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Hansen and Wright (1992)

- Hansen and Wright (1992) computed the above correlation with various data sources and two time periods. They found the correlation rages from 0.1 to -0.35 during 1955:3-1988:2, and rages from 0.07 to -0.14 during 1947:1-1991:3.
- All series are quarterly, are in 1982 dollars, logged and detrended with HP filter. The output series is the gross national product. Productivity is $\frac{y}{h}$.
- 4 hour series are used:
  Total hours in the household survey covers all industries.
  Total hours in the establishment survey covers only non-agricultural industries.
  Total hours in the household survey covers only non-agricultural industries.
  Hours worked in efficiency units.
- Data source: Citicorp's Citibase data bank and Hansen 1991.

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- U.S. quarterly data from 1948:1-1994:4.
- Productivity: real GDP/total civilian employment
- Employment: Total civilian employment.
- Hours: Total employee-hours in non-agricultural establishments.
- Data source: Citibase (DRI Basic Economics database).

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Data used in Figure Employment Volatility Smile

- Quarterly data from 1976 Q1 to 2019 Q4.
- **Real GDP**: obtained from FRED website, series GDPC1, Billions of Chained 2012 Dollars, Seasonally Adjusted.
- **Aggregate Employment to Population Ratio**: obtained from FRED website, series EMRATIO, Percent, Seasonally Adjusted.
- **By-Age Employment to Population Ratio**: calculated by using monthly CPS data from Integrated Public Use Microdata Series (IPUMS) and then deseasonalized by X-13 ARIMA-SEATS. The quarterly data is obtained by taking quarterly averages of monthly data.
- All the data are in logs and are are detrended by using HP filter with smoothing parameter 1600 before we compute the percentage standard deviation.

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Why We Use Employment to Population Ratio

- Galí (1999) tries two measures for labor input, “hours” and “civilian employment level”.
- In our replication of Galí (1999) and our employment curve in our MAR model, we use employment to population ratio as our measure for labor input for the following reasons:
  - (In draft) Most previous papers use hours instead of the extensive margin of employment, but, for example, Galí (1999) use an extensive margin measure in one of his robustness checks. We use an extensive margin because because we think it is at least as important as the intensive margin when discussing labor market outcomes.[2]
  - (In draft) We use the employment to population ratio in all our VARs (i) because of its interpretability and (ii) because a substantial amount of previous work used per capital variables when measuring labor market outcomes in Galí (1999)-type VARs (Christiano et. al (2000), “Involuntary unemployment and the business cycle”,“Review of Economic Dynamics”.)
  - We use employment to population ratio not employment level because we consider the employment across ages 16-65, thus it makes sense for us to consider the size of the population.

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More on FPCA Implementation

To map f_t to $\pi_m(f_t)$, choose a basis. We use a wavelet basis, and

- Demean the functional times f_t by removing the over time mean at each age.
- Perform wavelet decomposition of f_t at each time using a sufficiently large number of basis, say M, and collect wavelet coefficients into a $(T \times M)$ matrix W. We then have

$$W =< W, U_1 > U_1+ < W, U_2 > U_2 + \cdots + < W, U_M > U_M,$$

  where $< W, U_i >$ is the factor loading and U_i is the factor, for $i = 1, \cdots, M$.

We then apply the usual PCA to $W'W$, which captures the variations in our functional time series (w_t).

Finally we choose $m << M$ to approximate f_t using the cumulative scree plot. We may of course use other available methods, such as eigen value ratio tests, to determine m. We pick $m = 4$, which explains more than 85% of variations in the time series of the functional variable (f_t). back